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The Hill Theorem

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Abstract

It is studied how strong interactions in the Higgs sector can lead to deviations in vector boson selfcouplings. Normally such effects are small due to Veltman's screening theorem. It is shown that strong interactions are possible, if there is a hierarchy of strong interactions in the Higgs sector. This is called Hill's theorem.

1 Introduction

The standard model of the weak interactions gives a satisfactory description of all experiments up to date. The structure of the model is also elegant in the sense that the interactions are determined by the gauge principle and therefore of a geometric origin. The exception to this is the Higgs sector of the theory. The Higgs sector is responsible for the large number of free parameters, in casu the Yukawa couplings to the fermions of the model. The Yukawa couplings give rise to the masses of the theory via the mechanism of spontaneous symmetry breaking. There appears to exist essentially no pattern to the structure of the fermion masses. It seems therefore reasonable to ask whether the Higgs sector is fundamental. Possibly strong interactions could be present. Since an explicit model of such strong interactions is lacking, the

best one can do as a start is to study the theory without the Higgs explicitly present. If one removes the Higgs the theory becomes nonrenormalizable and cut-off dependent results appear. Such efffects can be estimated, for instance by taking the Higgs particle to be very heavy and calculating the radiative corrections to vectorboson properties [1]. Typically the generated effects are only logarithmic in the cut-off. Therefore no particularly strong interactions are present at low energy. This is known as Veltman's theorem. Since the divergences can be calculated directly in the non-renormalizable model [2,3], this is at first sight a universal feature. To study whether Veltman's theorem can be avoided one therefore has to complicate the standard model. A way to do this is to add a strongly interacting singlet sector to the theory. The interactions in the Higgs sector can be affected by the presence of the singlet sector. As a consequence the radiative corrections to vector boson physics can be changed. Explicit calculations show that the resulting interactions among the vector bosons can become strong. However for this to happen a hierarchy of strong couplings has to be present in the theory. This would appear to be a general feature, regardless of the exact nature of the strong interactions. Therefore I claim that the following holds (Hill's theorem) : "Veltman's screening theorem can be avoided if there is a hierarchy of strong interactions in the Higgs sector".

2 Removing the Higgs particle

The weak interactions are mediated by the exchange of massive vector bosons. Within the standard model the mass of the vector bosons is due to the Higgs mechanism. We are interested here in strongly interacting vector bosons where no physical Higgs is present. Therefore we are interested in ways to remove the Higgs particle from the theory. There are essentially two approaches possible. One is the traditional approach making the Higgs very heavy. Since the mass of the Higgs depends on the coupling making the Higgs heavy corresponds to a srongly coupled theory. The standard model is a gauged linear σ model with the Lagrangian:

$$\mathcal{L} = -\frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - \frac{\lambda}{8}(\Phi^\dagger\Phi - f^2)^2 \tag{1}$$

$$\Phi = (\sigma + i\vec{\tau} \cdot \vec{\phi}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

At the tree level, the Higgs particle can be removed by taking the limit $\lambda \to \infty$ or, equivalently, $m_H^2 \to \infty$. The standard model then turns into a gauged nonlinear σ-model

$$\mathcal{L} = \frac{f^2}{4} Tr(D_\mu U)^\dagger (D^\mu U) \tag{3}$$

$$U = \sqrt{1 - \vec{\pi}^2} + i\vec{\tau} \cdot \vec{\pi} \tag{4}$$

$$\vec{\pi} = \frac{\vec{\phi}}{f} \tag{5}$$

which is equivalent to massive Yang-Mills theory. It can be seen on the formal level that the standard model reduces to (3) in the limit $\lambda \to \infty$ by noticing that the potential acts like a constraint in this case.
A second [4], more speculative way to remove the Higgs boson relies on the possibility of having a large nonminimal coupling $\xi \Phi^\dagger \Phi R$ of the Higgs boson to gravity. As a consequence there is a large wave function renormalization of the Higgs boson by a factor $1/\sqrt{1+12\xi}$. This reduces the coupling of the Higgs boson to gravitational strength, so that the Higgs effectively disappears from the theory.
Given the removal of the Higgs boson from the theory there are two ways to perform the radiative corrections. One is to calculate directly in the non-linear model [2,3]. This calculation gives logarithmic divergences that show up as poles in $1/(n-4)$ in dimensional regularization. The other way is to calculate those physical effects in the linear model, that grow with the Higgs mass [1]. Only a logarithmic growth $log(m_H)$ is present in experimentally measurable quantities. These logaritmic effects are in one to one correspondence with the divergences in the non-linear model with the replacement

$log(m_H) = 1/(n-4)$. If one ignores the hypercharge field the effects can be summarized by the following effective Lagrangian

$$\mathcal{L}_{eff} = \alpha_1 Tr(V_\mu V^\mu) Tr(V_\nu V^\nu) + \\ \alpha_2 Tr(V_\mu V^\nu) Tr(V_\mu V^\nu) + g\alpha_3 Tr(F_{\mu\nu}[V^\mu, V^\nu]) \tag{6}$$

where

$$V_\mu = (D_\mu U)U^\dagger \tag{7}$$

and

$$F_{\mu\nu} = (\partial_\mu - \frac{ig}{2}\vec{W}_\mu \cdot \vec{\tau})\frac{\vec{W}_\nu \cdot \vec{\tau}}{2i} - (\mu \leftrightarrow \nu) \tag{8}$$

Explicit calculation in the linear model gives

$$\alpha_1 = \frac{1}{384\pi^2}\ln(m_H^2/M_W^2) + \mathcal{O}(1) \tag{9}$$

$$\alpha_2 = \frac{1}{192\pi^2}\ln(m_H^2/M_W^2) + \mathcal{O}(1) \tag{10}$$

$$\alpha_3 = -\frac{1}{384\pi^2}\ln(m_H^2/M_W^2) + \mathcal{O}(1) \tag{11}$$

The fact that these corrections are small, in particular that no correction growing like m_H^2 is present is called Veltman's screening theorem. In order to avoid this, one has to make changes in the Higgs sector.

3 Hill's theorem

In order to avoid the consequences of the screening theorem, changes to the theory are necessary. The absence of quadratic divergences is due to a delicate cancellation between radiative corrections and counterterms in the Higgs propagator. These cancellations can be removed by adding extra interactions in the Higgs sector. The simplest way to do this is to add a strongly interacting singlet to the model [5]. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - \frac{1}{2}(\partial_\mu x)^2 - \frac{\lambda_1}{8}(\Phi^\dagger\Phi - f_1^2)^2$$
$$-\frac{\lambda_2}{8}(2f_2x - \Phi^\dagger\Phi)^2 + \mathcal{L}_{gauge} \qquad (12)$$

Within this model we are interested in the limit $\lambda_2 >> \lambda_1 >> 0$. In this limit the effects of the extra field can in principle become strong, so that radiative effects can also feed down to the interactions in the vector boson sector. The result of the explicit calculation of the one loop radiative corrections is very simple if one takes the limit $f_2 >> f_1$ at the end of the calculation. In this limit the whole effect of the interactions beyond the standard model can be summarized by the parameter β:

$$\beta = 128\pi^2(\alpha_2 - 2\alpha_1) = \lambda_2/\lambda_1 \qquad (13)$$

The parameter β thus takes arbitrary values in this model. The same happens in an alternative model [6]. However in both cases large effects only appear when there is a hierarchy of strong interactions. This appears therefore to be a general feature. As a consequence one can claim the following (Hill's theorem): "Strong interactions can appear in the vector boson sector when there is a hierarchy of interaction strengths in the Higgs sector". One could object that the model looks unnatural. However one presumably should not consider the x field to be fundamental, but only as an effective description for underlying strong dynamics. In that case the theorem could play a role in technicolor dynamics. The parameter β is known from pion physics [7,8]. It is the parameter that is responsible for the formation of I=1 bound states. The consequence of strong interactions would be the formation of a I=1 bound state of vector bosons. In principle such a state could be seen at the Tevatron if it is light enough. Present limit are very weak, $\beta < 500$ [9]. As a rough estimate this corresponds to strong technicolor dynamics at a scale of about 20-50 TeV .

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